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1990 J. Phys.: Condens. Matter 2 4779

(http://iopscience.iop.org/0953-8984/2/21/014)

J. Phys.: Condens. Matter 2 (1990) 4779–4784. Printed in the UK

LETTER TO THE EDITOR

The hierarchy of fractional states: Haldane's scheme compared with composite particle approaches

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Received 15 March 1990

Abstract. In this letter, we point out that there may be a composite-boson approach for the fractional quantum Hall effect parallel to the composite-fermion approach, both of them exhibiting hierarchical structure. We then compare the composite-fermion and composite-boson approaches with Haldane's scheme.

It is well known that the fractional quantum Hall effect (FQHE) is a many-body phenomenon occurring in high-mobility two-dimensional electron systems at very low temperatures in high magnetic fields [1]. The FQHE is characterised by minima in the longitudinal resistivity ρ_{xx} and quantised Hall plateaus $\rho_{xy} = h/\nu e^2$ at Landau level filling factors $\nu = p/q$, where q is always odd in the lowest Landau level. Laughlin [2] has shown that the Coulomb interaction between electrons leads to new collective ground states at $\nu = 1/q$, which play a special role in the FQHE. The other fractional states, according to the hierarchical model [3], are derived from 'parent' states $1/q$, in which 'daughter' states are formed by correlation of the quasiparticles of the preceding state.

Very recently, Jain [4] has proposed a new composite-fermion approach for understanding the FQHE. He has shown that the FQHE of electrons with $\nu = p/(2mp \pm 1)$ can be physically understood as a manifestation of the integral quantum Hall effect (IQHE) of composite fermions with integer filling factor p, where a composite fermion consists of an electron bound to $2m$ flux quanta ($\varphi_0 = hc/e$). As noted by Jain [4], almost all the observed fractions are included in the filling factor $\nu = p/(2mp \pm 1)$ with $m = 0, 1, 2, \ldots$ and $p = 1, 2, 3, \ldots$.

In this letter, we first point out that the composite-fermion approach also exhibits hierarchical structure, and the filling factor $\nu = p/(2mp \pm 1)$ obtained by Jain [4] is the first level of the new hierarchy. (In fact, all fractions with odd denominators in the range $0 < \nu < 1$ are included in the new hierarchy.) Then, we present a composite-boson approach [5] for the FQHE parallel to the composite-fermion approach [4]. Finally, we compare Haldane's scheme with the composite-fermion and composite-boson approaches.

Following Haldane [3], if the parent state is one of the spinless-particle Laughlin states with $\nu = 1/q$, the first-level hierarchy equation is

$$N_\varphi = q(N-1) + \alpha N' \qquad (\alpha = \pm 1) \tag{1}$$

$$N = p_1(N'-1) \qquad (p = \text{even}). \tag{2}$$

In equation (1), N' denotes the number of quasiparticles ($\alpha = -1$) or quasiholes ($\alpha = +1$). Equation (2) is analogous to equation (1), except that (i) the number of independent single-electron states N_φ has been replaced by the number of independent quasiparticle states N, and (ii) p_1 must be even, because of the nominal Bose statistics of quasiparticles in Haldane's scheme. From equations (1) and (2), the filling factor $\nu = N/N_\varphi$ is $1/(m + \alpha/p_1)$ in the thermodynamic limit. The hierarchy of the FQHE of electrons is given by the continued fractions [3]

$$\nu_{\mathrm{H}}[m, \alpha_1 p_1, \ldots, \alpha_n p_n] = \cfrac{1}{m + \cfrac{\alpha_1}{p_1 + \cfrac{\alpha_2}{\ddots + \cfrac{\alpha_n}{p_n}}}} \tag{3}$$

where m is odd for a Fermi system and even for a Bose system, the p_i ($i = 1, 2, \ldots, n$) are even and the α_i ($i = 1, 2, \ldots, n$) are either 0 or ± 1. The subscript H denotes Haldane's scheme.

By analogy with equations (1) and (2), we have the first-level hierarchy equation for the IQHE of composite fermions:

$$N_\varphi = 2m(N_0 - 1) + \alpha_0 N \qquad (\alpha_0 = \pm 1) \tag{4}$$

$$N_0 = p_0(N - 1) \qquad (p_0 = \text{integer}). \tag{5}$$

In equation (4), N denotes the number of electrons ($\alpha_0 = \pm 1$ corresponds to magnetic field in the $\mp z$ direction). Equation (5) is analogous to equation (4), except that (i) the number of independent single-electron states N_φ has been replaced by the number of independent 'quasiparticle' states N_0, and (ii) p_0 must be a non-zero integer, since it denotes an integer filling factor in the composite-fermion approach. From equations (4) and (5), the filling factor $\nu = N_0/N_\varphi$ is $1/(2m + \alpha_0/p_0)$ in the thermodynamic limit [4]. Generally, the hierarchy of the IQHE of composite fermions can be expressed as

$$\nu_{\mathrm{CF}}[m, \alpha_0 p_0, \alpha_1 p_1, \ldots, \alpha_n p_n] = \cfrac{1}{2m + \cfrac{\alpha_0}{p_0 + \cfrac{\alpha_1}{p_1 + \cfrac{\alpha_2}{\ddots + \cfrac{\alpha_n}{p_n}}}}} \tag{6}$$

where $m = 0, 1, 2, \ldots$, $\alpha_0 = \pm 1$, the α_i ($i = 1, 2, \ldots, n$) are either 0 or ± 1, and $p_j = 1, 2, 3, \ldots$ ($j = 0, 1, 2, \ldots, n$). The subscript CF denotes the composite-fermion scheme. Equation (6) can be understood as a natural consequence of the fact that the 'quasiparticles' of the nth condensate play the role of flux quanta for the 'quasiparticles' of the $(n + 1)$th condensate. Unlike in Haldane's scheme, the 'quasiparticles' in the composite-fermion approach are, in fact, electrons. The experimentally observed states

4/11, 4/13 [4] and 5/7 [6] are the second level of equation (6), which can be expressed as

$$\nu_{\mathrm{CF}}[1, 1, 3] = 4/11 \qquad \nu_{\mathrm{CF}}[1, 1, -5] = 4/13 \qquad \nu_{\mathrm{CF}}[1, -2, -3] = 5/7.$$

I conjecture that there may exist a hierarchy of the IQHE of composite bosons, similar to equation (6), as follows:

$$\nu_{\mathrm{CB}}[m, \alpha_0 p_0, \alpha_1 p_1, \ldots, \alpha_n p_n] = \cfrac{1}{(2m+1) + \cfrac{\alpha_0}{p_0 + \cfrac{\alpha_1}{p_1 + \cfrac{\alpha_2}{\ddots + \cfrac{\alpha_n}{p_n}}}}} \tag{7}$$

where $m = 0, 1, 2, \ldots, \alpha_0 = \pm 1$, the α_i $(i = 1, 2, \ldots, n)$ are either 0 or ± 1, and $p_j = 1, 2, 3, \ldots$ $(j = 0, 1, 2, \ldots, n)$. The subscript CB denotes the composite-boson scheme.

Comparing equations (6) and (7) with equation (3), we find that there are some delicate distinctions between them.

(i) In equations (6) and (7), m is allowed to equal zero, whereas in equation (3) m must be non-zero. Therefore, the form of the first level of equations (6) and (7) is different from that of equation (3), as indicated by the broken boxes in equations (6), (7) and (3).

(ii) In equation (3), the p_i must be even, because of the nominal Bose statistics of quasiparticles in Haldane's scheme. However, in equations (6) and (7), the p_j are non-zero integers according to the gauge argument of Laughlin [7].

In figures 1(a) and 1(b), the predicted sequences of fractional states for the first level of equations (6) and (7) are shown, respectively. An experimentally confirmed order of stability of the fractions is illustrated in figure 1(a), i.e. a given fractional state in the right-hand (left-hand) half is more stable than the one directly above it and the one on its right (left) [4]. This situation may be understood as follows. For a fixed p_0, an electron is difficult to bind to more flux quanta $2m\varphi_0$ ($\varphi_0 = hc/e$), since a collapse of the gap due to an 'unbinding transition' is more likely for larger values of m. On the other hand, for a fixed m, the correlations between 'quasiparticles' are weaker for larger filling factor p_0. From figure 1(a), we see that the primary even-denominator fractions 1/2, 1/4, . . . are the limits of fractional sequences with odd denominators. This probably means that the fractional states with even denominators have the weakest correlation in the composite-fermion approach. Hence, those fractional states are hard to observe experimentally. However, the primary odd-denominator fractions 1/3, 1/5, . . . are the limits of even/odd-denominator alternating fractional sequences as shown in figure 1(b). It should be noted that if one can find a system in which the condition described in the composite-boson scheme [5] is satisfied, then we can expect that in such a system the primary even-denominator fractions 1/2, 1/4, . . . are readily observable.

In figures 2(a) and (b), the predicted sequences of fractional states deriving from the 1/3 and the 1/5 parent states are shown, respectively. We have chosen the $p_i = 2$ in equation (3). The fractions that have been observed experimentally are denoted by

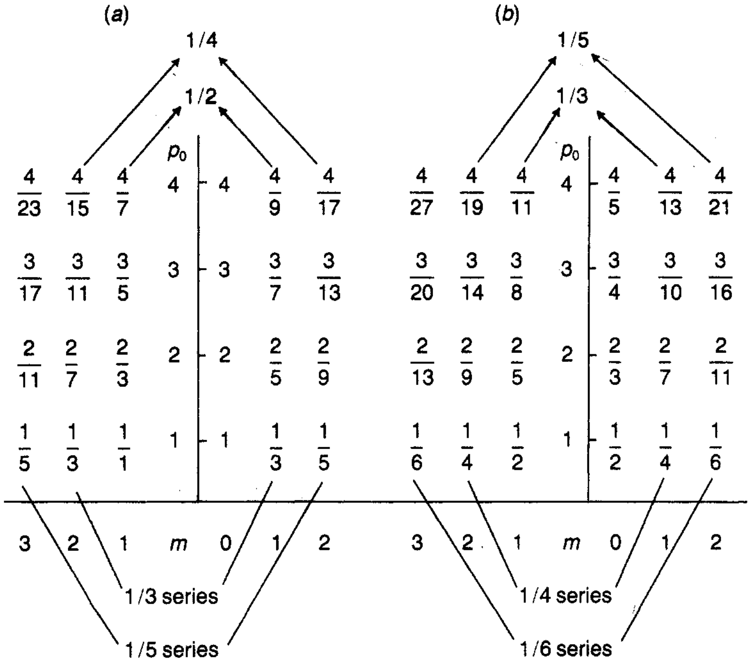

Figure 1. (*a*) The fractions for the first level of equation (6) given by $p_0/(2mp_0+1)$ and $p_0/(2mp_0-1)$ are shown in the right-hand half and the left-hand half, respectively. The primary even-denominator fractions 1/2 and 1/4 are the limits of fractional sequences $p_0/(2p_0 \pm 1)$ (for $m=1$) and $p_0/(4p_0 \pm 1)$ (for $m=2$), respectively. The 1/3 (1/5) series, i.e. $p_0/(2p_0+1)$ and $p_0/(4p_0-1)$ ($p_0/(4p_0+1)$ and $p_0(6p_0-1)$) for $p_0 = 1, 2, 3, \ldots$, just correspond to those fractions on the 1/3 (1/5) Haldane tree as shown in figure 2(*a*) (figure 2(*b*)). (*b*) As (*a*) but for the first level of equation (7). The primary odd-denominator fractions 1/3 and 1/5 are the limits of fractional sequences $p_0/(3p_0 \pm 1)$ (for $m=1$) and $p_0/(5p_0 \pm 1)$ (for $m=2$), respectively. The 1/4 (1/6) series, i.e. $p_0/(3p_0+1)$ and $p_0/(5p_0-1)$ ($p_0/(5p_0+1)$ and $p_0(7p_0-1)$) for $p_0 = 1, 2, 3, \ldots$, just correspond to those fractions on the 1/4 (1/6) Haldane tree, which is not illustrated explicitly.

asterisks. We term all the fractional states deriving from the $1/q$ (q is either odd or even) parent state a '$1/q$ Haldane tree'. As illustrated in figures 2(*a*) and (*b*), all the observed fractions that belong to different hierarchical levels are located *on* the 'surface' of Haldane trees, and the fractions located *in* the Haldane trees are not observed experimentally. In Haldane's scheme, it is not clear why the fractions *in* the Haldane trees are more difficult to observe than those fractions *on* the Haldane trees with higher hierarchical levels. For example, in the 1/3 Haldane tree, the 3/7, the 4/9, the 5/11 and the 6/13 states are observed, whereas the 5/13 state, which is one of the daughter states of the 2/5 state and is located *in* the Haldane tree as shown in figure 2(*a*), is not observed experimentally.

Comparing figure 1(*a*) with figure 2, we find that fractions in the 1/3 and the 1/5 series (figure 1(*a*)) correspond to those fractions *on* the 1/3 and the 1/5 Haldane trees (figures 2(*a*) and (*b*)), respectively. We also find that the fractions in higher hierarchical levels in the composite-fermion scheme (equation (6)) generally correspond to those fractions *in* the Haldane trees. For example, the $\nu_{CF}[1, 2, -3] = 5/13$ state, which is located *in* the 1/3 Haldane tree, is a second-level state of equation (6). Moreover, the

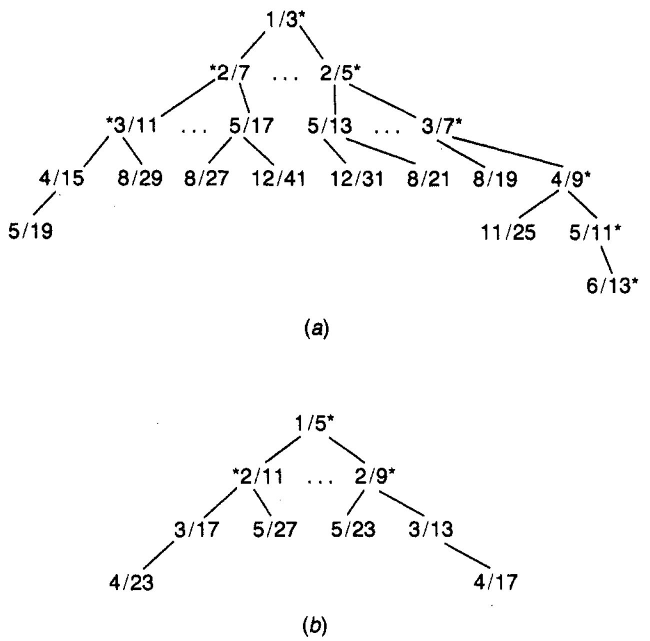

Figure 2. The 1/3 and the 1/5 Haldane trees, which are derived from equation (3) with the $p_i = 2$, are shown in (a) and (b), respectively. The asterisks indicate the observed fractions. The ellipses indicate the existence of additional intermediate fractions. For example, the 4/13 and the 4/11 states, which are derived from equation (3) with $p_1 = 4$, exist between the 2/7 and the 2/5 states in (a).

fractions belonging in higher hierarchical levels of equation (6) are located in positions nearer the centres of Haldane trees. For example, the $\nu_{\rm CF}[1, 3, -3] = 8/19$ and the $\nu_{\rm CF}[1, 2, -3, -2] = 8/21$ states are the second- and the third-level states of equation (6) [8], respectively. In the 1/3 Haldane tree, the 8/21 state is in a position nearer the centre of the tree than the 8/19 state, as shown in figure 2(a). From this discussion, we can understand why the fractions *on* the Haldane trees are more readily observed than those fractions *in* the Haldane trees. One should note that the fractions for $p_i \geqslant 4$ in Haldane's scheme (equation (3)) generally correspond to those fractions in higher levels of equation (6). For example,

$$\nu_{\rm H}[3, -4] = \nu_{\rm CF}[1, 1, 3] = 4/11 \qquad \nu_{\rm H}[3, 4] = \nu_{\rm CF}[1, 1, -5] = 4/13.$$

The same argument can be repeated in comparing the composite-boson scheme (equation (7)) with the $1/q$ (q is even) Haldane trees. However, there has been no experimental evidence to support this argument until recently.

In summary, we have pointed out there may be a composite-boson approach for the FQHE similar to the composite-fermion approach proposed by Jain, both of them exhibiting hierarchical structure. We have shown that the composite-particle approaches have the merit of helping us in understanding why some fractions are more readily observed than others. We believe that the discussion presented here is useful to the further understanding of both the relationship between various fractional states and the connection between different theoretical approaches.

References

[1] For experiments on the fractional quantum Hall effect see, for example,
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[8] In the hierarchical model, a state in a level can also belong in the levels higher than that level. We term a state an ith-level state in the sense that the lowest level this state belongs to is the ith level. For example, $\nu_{\mathrm{CF}}[1, 2] = \nu_{\mathrm{CF}}[1, 1, 1] = \ldots = 2/5$; the 2/5 state is a first-level state of equation (6).